Computational Models of Argument
C. Reed et al. (Eds.)

doi:10.3233/FAIA240316

Attack-Defense Semantics of Argumentation

Beishui LIAO [a,1] and Leendert VAN DER TORRE [b]
[a] *Zhejiang University*
[b] *University of Luxembourg*

Abstract. Abstract argumentation is an important research area in AI. It is mainly about the acceptability of arguments in an argumentation framework. The classical notion of defense has not fully reflected some useful information implicitly encoded by the interaction relation between arguments. In this paper, instead of using arguments and attacks as first citizens, a novel notion of attack-defense is adopted as a first citizen, based on which a theory of attack-defense framework and attack-defense semantics are established, where an attack-defense is a triple (x, y, z), meaning that: an argument x defends an argument z against an attacker y. Attack-defense semantics can be used not only to identify the impact of arguments in some odd cycles, and remove some "useless" defenses, but also to capture new types of equivalence that cannot be represented by the existing notions of equivalence of argumentation frameworks. In addition, it shows that an attack-defense framework and attack-defense semantics can represent some knowledge that cannot be represented in Dung-style argumentation, e.g., some context-sensitive knowledge in a dialogue.

Keywords. Dung's abstract argumentation, equivalence, expressiveness, odd cycles

1. Introduction

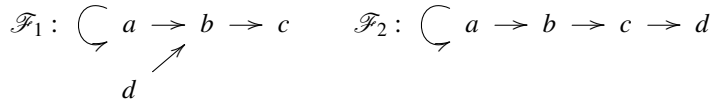
In the field of AI, abstract argumentation is mainly about the acceptability of arguments in an argumentation framework (AF) [9,12]. A set of arguments that is collectively acceptable according to some criteria is called an extension. There are two basic criteria for defining all kinds of extensions that are based on the notion of admissible set, called conflict-freeness and defense. An argument is defended by a set of arguments if every attacker of this argument is attacked by at least one argument in this set. Obviously, the notion of defense plays an important role in evaluating the status of arguments. However, this usage of the classical notion of defense has not fully reflected some useful information implicitly encoded by the interaction relation between arguments. The latter can be used to deal with some important problems in formal argumentation.

The first problem is about the expressivity of Dung-style argumentation. In many situations, the combination of arguments and attacks to form defenses is contextual and may not refer to an AF. For instance, in a dialogue, when a proponent says c and an opponent says b, the proponent uses a as a counterargument. Then, the proponent says e and the opponent says b again. This time, the proponent uses d as a counterargument,

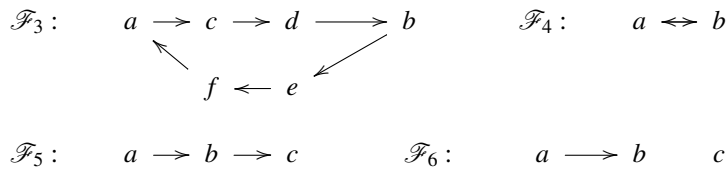
[1] Corresponding Author: Beishui Liao.

rather than a, due to some reason, say, rhetoric. It is interesting to note that such context-sensitive knowledge cannot be represented by Dung-style argumentation.

The second problem is about the treatment of odd cyles. Intuitively, arguments in some odd cycles cannot provide suppport for the acceptance of some other arguments. In $\mathscr{F}_1$, argument a is not useful for the acceptance of c, while in $\mathscr{F}_2$, argument a makes all other argument not acceptable. Is it possible to explicitly represent the impact of such arguments on other arguments, and remove some "useless" impact?



The third problem is about the equivalence between AFs. Intuitively, some AFs are equivalent in terms of some interpretations, but cannot be represented by the existing notions of equivance. Some of AFs are not equivalent but cannot be well captured by the existing notions of equivance. For instance, $\mathscr{F}_3$ and $\mathscr{F}_4$ are equivalent w.r.t. the acceptance of a and b, in the sense that in both AFs, a is the root reason to accept a, and b is the root reason to accept b. $\mathscr{F}_5$ and $\mathscr{F}_6$ are equivalent according to the notion of standard equivalence, but the reasons for accepting c in the two AFs are different.



To address the above problems, instead of using arguments and attacks as first citizens, we adopt a novel notion of attack-defense as a first citizen and establish a theory of an attack-defense framework and attack-defense semantics, where an attack-defense is a triple (x,y,z), meaning that: argument x defends argument z against attacker y. Since a successful attack-defense contains not only accepted arguments but also the information about why arguments are accepted, an attack-defense extension contains more information than that of the corresponding argument extension.

The structure of this paper is as follows. In Section 2, we define an attack-defense framework and attack-defense semantics. In Section 3, we formulate some important properties of attack-defense semantics. In Section 4, we study attack-defense equivalence. In Section 5, attack-defense semantics in Dung-style argumentation is presented. In section 6, we conclude the paper. Due to space limit, we omit all proofs in this paper. Please refer to the following link [2] for details.

2. Attack-defense framework and attack-defense semantics

In this section, we introduce notions of attack-defense framework and attack-defense semantics. An attack-defense is a triple (x,y,z), in which x, y and z are arguments. To generalize the notion of attack-defense such that every argument has at least one attack-defense, we introduce two special arguments, denoted $\top$ and $\bot$ (slightly abusing notations), indicating that they are always accepted and rejected, respectively. So, in (x,y,z),

[2] https://github.com/CYLsylvia/ZLARE/blob/main/Appendix.pdf

z is an argument other than $\top$ and $\bot$. Given that z is not $\bot$, y cannot be $\top$. Otherwise, z is $\bot$. By $(\top,\bot,z)$, we say that z is defended by $\top$ against $\bot$. By $(\bot,y,z)$, we say that z is defended by $\bot$ against y. Since $\bot$ cannot be accepted in any case, $(\bot,y,z)$ also cannot be successful in any case. The notion of attack-defense shares some similarity with the notion of weak-defence used in [10] in the sense that the relation of one argument (partially) defending another argument is explicitely represented and in the new semantics. The difference is that in [10], an attack-defense is a tuple in the form of $\langle x,z\rangle$, where z's attacker is not included, while in this paper, an attack-defense is a triple (x,y,z), where z's attacker y is included. Note that this fundamental defference has the following significant consequences. First, weak-defence is viewed as a type of argument, and no new semantics based on this notion is defined, while attack-defense is used to defined a new semantics. Second, the semantics based on the notion of attack-defense has some interesting properties that will be introduced in Section 3.

Formally, we have the following definition.

Definition 1 (Attack-defense framework)**.** *Let U be the universe of arguments, which contains $\top$ and $\bot$. An attack-defense framework is a set of attack-defenses $T \subseteq U \times U \setminus \{\top\} \times U \setminus \{\top,\bot\}$. We use z^x_y to denote that x is a defender of a defendee z against an attacker y.*

Given an attack-defense z^x_y, we use the following notations to denote the defendee, defender and attacker in the attack-defense: $\mathbf{defendee}(z^x_y) = z$, $\mathbf{defender}(z^x_y) = x$, and $\mathbf{attacker}(z^x_y) = y$. Given a set of attack-defenses $D \subseteq T$, we use the following notations: $\mathbf{defendee}(D) = \{z \mid z^x_y \in D\}$, $\mathbf{defender}(D) = \{x \mid z^x_y \in D\}$, $\mathbf{attacker}(D) = \{y \mid z^x_y \in D\}$, and $\mathbf{argument}(D) = \mathbf{defendee}(D) \cup \mathbf{defender}(D) \cup \mathbf{attacker}(D)$.

Then, an attack-defense semantics defines sets of attack-defenses that all are successful together.

Definition 2 (Attack-defense semantics)**.** *Let U be the universe of arguments, $U' = U \setminus \{\top\}$, and $U'' = U \setminus \{\top,\bot\}$. An attack-defense semantics is defined as a partial function $\Sigma : 2^{U\times U'\times U''} \to 2^{2^{U\times U'\times U''}}$, which associates a set of attack-defenses with a set of subsets of these attack-defenses.*

Intuitively, we say that an attack-defense (x,y,z) is successful w.r.t. a set of attack-defenses D if x is $\top$ or a defendee of D, and for each $y' \neq y$ that attacks z, there is an attack-defense (x',y',z) in D for some $x' \in \mathbf{defendee}(D)$.

Definition 3 (Successful attack-defense)**.** *Let $D \subseteq T$ be a set of attack-defenses, and $z^x_y \in T$ be an attack-defense. We say that z^x_y is successful w.r.t. D if $x = \top$ or $x \in \mathbf{defendee}(D)$, and for each $y' \neq y$ that attacks z, $\exists z^{x'}_{y'} \in D$ for some $x' \in \mathbf{defendee}(D)$.*

We say that a set of attack-defenses D is admissible iff every attack-defense in D is successful w.r.t. D, and no argument that is both a defendee and an attacker in D.

Definition 4 (Admissible set of attack-defenses)**.** *$D \subseteq T$ is admissible iff every attack-defense in D is successful w.r.t. D, and $\mathbf{defendee}(D) \cap \mathbf{attacker}(D) = \emptyset$.*

Definition 5 (Complete attack-defense extension)**.** *D is complete iff D is admissible, and for every attack-defense in T, if it is successful w.r.t. D, then it is in D.*

Definition 6 (Preferred attack-defense extension)**.** *D is preferred iff D is a maximal complete attack-defense extension w.r.t. set inclusion.*

Definition 7 (Stable attack-defense extension)**.** *D is stable iff D is admissible, and for all* $z_y^x \in T \setminus D$, $y \in \mathbf{defendee}(D)$.

Theorem 1. *If D is a stable attack-defense extension, then it is complete and preferred, but not necessarily vice versa.*

Example 1. *Let* $T_1 = \{c_b^a, b_a^\perp, a_\perp^\top, e_b^d, b_d^\perp, d_\perp^\top\}$. *There is only one attack-defense extension under all semantics* $D = \{a_\perp^\top, d_\perp^\top, c_b^a, e_b^d\}$.

Similar to Dung's theory, attack-defense semantics can also be characterized by a function.

Definition 8 (Characteristic function)**.** *The characteristic function, denoted by* F_T*, of an attack-defense framework T is defined as follows:*

$$F_T : 2^T \to 2^T,$$
$$F_T(D) = \{z_y^x \mid z_y^x \text{ is successful w.r.t. } D\}.$$

Example 2. *Let* $T_2 = \{b_a^\top, c_f^e, e_d^c\}$. *We have:* $F_{T_2}(\emptyset) = \{b_a^\top\}, F_{T_2}^2(\emptyset) = F_{T_2}(\emptyset)$. $F_{T_2}(\{c_f^e, e_d^c\}) = \{b_a^\top, c_f^e, e_d^c\}, F_{T_2}^2(\{c_f^e, e_d^c\}) = F_{T_2}(\{c_f^e, e_d^c\})$.

Definition 9 (Grounded attack-defense extension)**.** *The grounded attack-defense extension of T is the least fixed point of* F_T.

It is easy to see that F_T is monotonic w.r.t. set inclusion.

Theorem 2. *Let D be a set of attack-defenses. If* $\mathbf{defendee}(D) \cap \mathbf{attacker}(D) = \emptyset$*, then D is a complete attack-defense extension iff* $D = F_T(D)$. *The grounded attack-defense extension is the least complete attack-defense extension of F (w.r.t. set inclusion).*

In this paper we use $\Sigma(T)$ to denote the set of attack-defense extensions of T under semantics Σ, where $\Sigma \in \{CO, PR, GR, ST\}$ denotes respectively complete, preferred, grounded, and stable attack-defense semantics.

3. Properties of attack-defense semantics

Now, let us consider some properties of the attack-defense semantics.

The first property formulated in Theorem 3 is about the closure of attack-defenses: If both z_y^x and w_v^u are in an attack-defense extension, and $z_{y'}^w$ is an attack-defense for some argument $y' \in \mathbf{argument}(T)$, then $z_{y'}^w$ is also in the same attack-defense extension.

Theorem 3. *Let T be an attack-defense framework. For all* $D \in \Sigma(T)$, $x, u \in \mathbf{argument}(T)$, *if* z_y^x, $w_v^u \in D$, *then for some* $y' \in \mathbf{argument}(T)$, *if* $z_{y'}^w \in T$ *then* $z_{y'}^w \in D$.

The second property formulated in Theorem 4 is about the justfiability of attack-defenses: If z_y^x is in an attack-defense extension and $x \neq \top$, then there must be some $x_{y'}^u$ in the same attack-defense extension.

Theorem 4. *For all $D \in \Sigma(T)$, if $z_y^x \in D$ and $x \neq \top$ then there exists $x_{y'}^u \in T$ for some $u, y' \in$ **argument**(T), s.t. $x_{y'}^u \in D$.*

The third property is about the incompleteness of an attack-defense framework.

Theorem 5. *For all $z_y^x, v_y^u \in T$, it is not necessary that $v_y^x \in T$.*

This property can be illustrated by Example 1. When c_b^a and e_b^d are in T, but e_b^a and c_b^d are not in T.

The fourth property is about the unsatisfiability of some types of attack-defenses.

Definition 10 (Unsatisfiability of attack-defense)**.** *We say that z_y^x is unsatisfiable under semantics Σ iff z_y^x cannot be in any attack-defense extension under semantics Σ, where $\Sigma \in \{CO, PR, GR, ST\}$.*

In this paper, as typical examples, we introduce three types of unsatisfiable attack-defenses. The first type is the attack-defenses in the form $z_y^\perp$, which is by definition unsatisfiable.

Theorem 6. *Defense $z_y^\perp \in T$ is unsatisfiable under semantics $\Sigma \in \{CO, PR, GR, ST\}$.*

The second type is the attack-defenses related to self-attacking arguments.

Theorem 7. *Defenses $z_y^y, z_z^x \in T$ are unsatisfiable. Furthermore, if $z_y^y \in T$, then for all $u, v \in$ **argument**(T), u_v^y is unsatisfiable under semantics $\Sigma \in \{CO, PR, GR, ST\}$.*

The third type is the attack-defenses related to a 3-cycle consisting of x, y and z such that x attacks y, y attacks z, and z attacks x.

Theorem 8. *If there exist $x_z^y, y_x^z, z_y^x \in T$, then x_z^y, y_x^z and z_y^x are unsatisfiable under semantics $\Sigma \in \{CO, PR, GR, ST\}$.*

Given an attack-defense framework T, the sets of unsatisfiable attack-defenses depicted in Theorems 6, 7 and 8, are denoted $u_1(T)$, $u_2(T)$ and $u_3(T)$, respectively. Let $u(T) = u_1(T) \cup u_2(T) \cup u_3(T)$.

When an attack-defense is unsatisfiable under a semantics Σ, it might be removed from an attack-defense framework without affecting the evaluation of the status of other attack-defenses in the theory.

Under stable attack-defense semantics, the removal of an attack-defense might change the emptiness of the set of extensions of an attack-defense framework. For instance, let $T = \{z_y^\perp\}$, which has no stable attack-defense extension. However, after $z_y^\perp$ is removed, $T' = \{\}$ has a stable attack-defense extension, which is the empty set.

Under other semantics, we have the following theorem, which cannot be applied to stable attack-defense semantics.

Definition 11 (Reduct of an attack-defense framework)**.** *For any attack-defense framework T, the reduct of T is defined as $T^- = T \setminus u(T)$.*

Theorem 9. *For any attack-defense framework T, and for $\Sigma \in \{CO, PR, GR\}$, $\Sigma(T) = \Sigma(T^-)$.*

4. Attack-defense equivalence

We differentiate two types of attack-defense equivalence: standard equivalence and root equivalence.

Definition 12 (Standard attack-defense equivalence)**.** *Let T_1 and T_2 be two attack-defense frameworks. T_1 and T_2 are equivalent under attack-defense semantics Σ, denoted $T_1 \equiv_d^{\Sigma} T_2$, iff $\Sigma(T_1) = \Sigma(T_2)$.*

Root equivalence is defined in terms of the notion of transitive closure of defenses.

In this paper, if z_y^x and v_u^z are all in D, we say that x is an indirect defender of v w.r.t. D. For simplicity, when we consider the defense relation between x and z, we write (x,z), instead of (x,y,z) or z_y^x. The defense relation is transitive, i.e., if (x,z) and (z,u) hold, then (x,u) holds. Formally, we have the following definition.

Definition 13 (Transitive closure of defenses)**.** *For all $D \in \Sigma(T)$, let $\overline{D} = \{(x,z) \mid z_y^x \in D\}$. The transitive closure of $\overline{D}$ is denoted $\overline{D}^+$.*

Given an attack-defense extension D, for any argument $z \in \textbf{argument}(T)$, we say an argument $x \in \textbf{argument}(D)$ is a root reason of accepting z, if $(x,x) \in \overline{D}^+$ and $(x,z) \in \overline{D}^+$. We say that $\top$ is a root reason of accepting z, if $(\top,z) \in \overline{D}^+$.

Example 3. *Let $T_3 = \{a_f^e, b_d^c, c_a^f, d_c^a, e_b^d, f_e^b\}$. Under preferred attack-defense semantics, $PR(T_3) = \{D_1, D_2\}$, where $D_1 = \{a_f^e, d_c^a, e_b^d\}$ and $D_2 = \{b_d^c, f_e^b, c_a^f\}$. So, we have: $\overline{D_1} = \{(a,d),(d,e),(e,a)\}$. $\overline{D_1}^+ = \overline{D_1} \cup \{(a,a),(d,d),(e,e),(a,e),(d,a),(e,d)\}$. $\overline{D_2} = \{(b,f),(f,c),(c,b)\}$. $\overline{D_2}^+ = \overline{D_2} \cup \{(b,b),(c,c),(f,f),(b,c),(c,f),(f,b)\}$. So, in D_1, a, d and e are root reasons of accepting a, d and e, resp. The results are similar in D_2.*

Definition 14 (Root reasons for accepting arguments)**.** *Given an attack-defense extension D, for any argument $z \in \textbf{argument}(T)$, the root reasons for accpecting z, denoted $r(z,D)$, is defined as follows.*

$$r(z,D) = \{x \mid (x,x) \in \overline{D}^+, (x,z) \in \overline{D}^+\} \cup \{\top \mid (\top,z) \in \overline{D}^+\} \tag{1}$$

The set of root reasons for accepting arguments in T under semantics Σ is defined as follows.

$$root_{\Sigma}(z,T) = \{r(z,D) \mid D \in \Sigma(T)\} \tag{2}$$

Definition 15 (Root equivalence)**.** *Let T and T' be two attack-defense frameworks. For all $B \subseteq \textbf{argument}(T) \cap \textbf{argument}(T')$, we say that T and T' are equivalent w.r.t. the root reasons for accepting B under semantics Σ, denoted $T \equiv_{r,B}^{\Sigma} T'$, iff for all $z \in B$, $root_{\Sigma}(z,T) = root_{\Sigma}(z,T')$.*

When $B = \textbf{argument}(T) = \textbf{argument}(T')$, we write $T \equiv_r^{\Sigma} T'$ for $T \equiv_{r,B}^{\Sigma} T'$.

Example 4. *Continue Example 3. Let $T_3' = \{a_b^a, b_a^b\}$. $PR(T_3') = \{D_1', D_2'\}$, where $D_1' = \{a_b^a\}$, $D_2' = \{b_a^b\}$. Let $B = \{a,b\}$. Under preferred attack-defense semantics, it holds that $T_3 \equiv_{r,B}^{PR} T_3'$, because $root_{PR}(a,T_3) = root_{PR}(a,T_3') = \{\{a\},\{\}\}$, and $root_{PR}(b,T_3) = root_{PR}(b,T_3') = \{\{\},\{b\}\}$.*

Theorem 10. *For any attack-defense frameworks T and T', for any $B \subseteq$ **argument**$(T) \cap$ **argument**(T'), if $T \equiv_d^{\Sigma} T'$ then $T \equiv_{r,B}^{\Sigma} T'$, but not vice versa.*

5. Attack-defense semantics in Dung-style argumentation

In this section, we introduce the application of attack-defense semantics to Dung-style argumentation. First, let us recall some notions of Dung-style argumentation.

5.1. Preliminaries

According to Dung-style argumentation [9], an AF is defined as $\mathscr{F} = (\mathscr{A}, \rightarrow)$, where $\mathscr{A}$ is a set of arguments and $\rightarrow \subseteq \mathscr{A} \times \mathscr{A}$ is a set of attacks between arguments.

Let $\mathscr{F} = (\mathscr{A}, \rightarrow)$ be an AF. Given a set $B \subseteq \mathscr{A}$ and an argument $\alpha \in \mathscr{A}$, B attacks α, denoted $B \rightarrow \alpha$, iff there exists $\beta \in B$ such that $\beta \rightarrow \alpha$. We call an argument an *initial* argument if it has no attacker.

Given $\mathscr{F} = (\mathscr{A}, \rightarrow)$ and $E \subseteq \mathscr{A}$, we say: E is *conflict-free* if $\nexists \alpha, \beta \in E$ such that $\alpha \rightarrow \beta$; $\alpha \in \mathscr{A}$ is *defended* by E if $\forall \beta \rightarrow \alpha$, $E \rightarrow \beta$; B is *admissible* if E is conflict-free, and each argument in E is defended by E; E is a *complete extension* of $\mathscr{F}$ if E is admissible, and each argument in $\mathscr{A}$ that is defended by E is in E. E is a *preferred extension* of $\mathscr{F}$ if E is an maximal complete extension of $\mathscr{F}$. E is the *grounded extension* of $\mathscr{F}$ if E is the minimal complete extension of $\mathscr{F}$. W use $\sigma(\mathscr{F})$ to denote the set of σ extensions of $\mathscr{F}$, where $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}\}$ (indicating complete, preferred, grounded, stalbe semantics, reps.) is a function mapping each AF to a set of σ extensions, called σ semantics.

For AFs $\mathscr{F}_1 = (\mathscr{A}_1, \rightarrow_1)$ and $\mathscr{F}_2 = (\mathscr{A}_2, \rightarrow_2)$, we use $\mathscr{F}_1 \cup \mathscr{F}_2$ to denote $(\mathscr{A}_1 \cup \mathscr{A}_2, \rightarrow_1 \cup \rightarrow_2)$. The standard equivalence and strong equivalence of AFs are defined as follows.

Definition 16 (Standard and strong equiv. of AFs)**.** *[11] Let $\mathscr{F}$ and $\mathscr{G}$ be two AFs.*

- *$\mathscr{F}$ and $\mathscr{G}$ are of standard equivalence w.r.t. a semantics σ, in symbols $\mathscr{F} \equiv^{\sigma} \mathscr{G}$, iff $\sigma(\mathscr{F}) = \sigma(\mathscr{G})$.*
- *$\mathscr{F}$ and $\mathscr{G}$ are of strong equivalence w.r.t. a semantics σ, in symbols $\mathscr{F} \equiv_s^{\sigma} \mathscr{G}$, iff for all AF $\mathscr{H}$, it holds that $\sigma(\mathscr{F} \cup \mathscr{H}) = \sigma(\mathscr{G} \cup \mathscr{H})$.*

Example 5. *Consider $\mathscr{F}_3 - \mathscr{F}_6$ in Section 1. In terms of Definition 16, under complete semantics, we have: $\mathscr{F}_3 \not\equiv^{\mathrm{co}} \mathscr{F}_4$, $\mathscr{F}_3 \not\equiv_s^{\mathrm{co}} \mathscr{F}_4$; $\mathscr{F}_5 \equiv^{\mathrm{co}} \mathscr{F}_6$, $\mathscr{F}_5 \not\equiv_s^{\mathrm{co}} \mathscr{F}_6$.*

Given an AF $\mathscr{F} = (\mathscr{A}, \rightarrow)$, the kernel of $\mathscr{F}$ under complete semantics, call *c-kernel*, is defined as follows.

Definition 17 (c-kernel of an AF)**.** *[11] For an AF $\mathscr{F} = (\mathscr{A}, \rightarrow)$, the c-kernel of $\mathscr{F}$ is defined as $\mathscr{F}^{\mathrm{ck}} = (\mathscr{A}, \rightarrow^{\mathrm{ck}})$, where*

$$\rightarrow^{\mathrm{ck}} = \rightarrow \setminus \{\alpha \rightarrow \beta \mid \alpha \neq \beta, \alpha \rightarrow \alpha, \beta \rightarrow \beta\} \tag{3}$$

According to [11], it holds that $\mathrm{co}(\mathscr{F}) = \mathrm{co}(\mathscr{F}^{\mathrm{ck}})$, and for any AFs $\mathscr{F}$ and $\mathscr{G}$: $\mathscr{F}^{\mathrm{ck}} = \mathscr{G}^{\mathrm{ck}}$ iff $\mathscr{F} \equiv_s^c \mathscr{G}$.

5.2. Attack-defense framework of an AF

Given $\mathscr{F} = (\mathscr{A}, \rightarrow)$, the attack-defense framework of $\mathscr{F}$ can be defined in the following way.

For $x, y, z \in \mathscr{A}$, if $x \rightarrow y$ and $y \rightarrow z$, then (x, y, z) is an attack-defense. For each initial argument $z \in \mathscr{A}$, there is a unique defense $(\top, \bot, z)$. For an argument z that is attacked by an initial argument y, there is an attack-defense $(\bot, y, z)$.

Definition 18 (Attack-defense framework of an AF)**.** *Let $\mathscr{F} = (\mathscr{A}, \rightarrow)$ be an AF. The attack-defense framework of $\mathscr{F}$, denoted* $\mathrm{d}(\mathscr{F})$*, is represented as follow.*

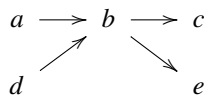
$$\mathrm{d}(\mathscr{F}) = \{(z_y^x \mid (x, y, z \in \mathscr{A}) \wedge (x \rightarrow y) \wedge (y \rightarrow z)\} \cup \{z_\bot^\top \mid (z \in \mathscr{A}) \wedge (z^- = \emptyset)\}$$
$$\cup \{z_y^\bot \mid (y, z \in \mathscr{A}) \wedge (y \rightarrow z) \wedge (y^- = \emptyset)\}$$

According to the definition of the attack-defense framework of an AF, we have the following theorem.

Theorem 11. *Let* $\mathrm{d}(\mathscr{F})$ *be the attack-defense framework of an AF $\mathscr{F} = (\mathscr{A}, \rightarrow)$. For all $z_y^x, v_y^u \in \mathrm{d}(\mathscr{F})$, it holds that $v_y^x \in \mathrm{d}(\mathscr{F})$.*

Example 6. *Continue Example 1.* $\mathrm{d}(\mathscr{F}_7) = T_1 \cup \{e_b^a, c_b^d\} = \{c_b^a, b_a^\bot, a_\bot^\top, e_b^d, b_d^\bot, d_\bot^\top, e_b^a, c_b^d\}$.

$\mathscr{F}_7$:



This example shows that the attack-defense framework in Example 1 cannot be represented as an attack-defense framework of an AF, in that e_b^a and c_b^d do not exist in T_1, but have to be included in $\mathrm{d}(\mathscr{F}_7)$. The underlying reason is that in practical dialogues, the combination of arguments and attacks to form defense are contextual and may not refer to an AF. However, in some situations, such kind of knowledge exists, as described in the first section.

5.3. Correspondence to Dung's semantics

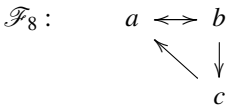
First, under a given attack-defense semantics, for each attack-defense extension of the attack-defense framework of an AF, the set of defendees of the attack-defense extension is an argument extension under a corresponding Dung's semantics.

Theorem 12. *For all $D \in \Sigma(\mathrm{d}(\mathscr{F}))$,* **defendee**$(D) \in \sigma(\mathscr{F})$*, where $\Sigma \in \{\mathrm{CO}, \mathrm{PR}, \mathrm{GR}, \mathrm{ST}\}$ and $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}\}$.*

Second, under a given Dung's semantics, for each extension E of an AF, there is a set of attack-defenses constructed from E such that this set is an attack-defense extension of the attack-defense framework generated from the AF, under a corresponding attack-defense semantics.

Theorem 13. *For all $E \in \sigma(\mathscr{F})$, let* $\mathrm{def}(E) = \{z_y^x \mid z_y^x \in \mathrm{d}(\mathscr{F}) : x, z \in E\} \cup \{z_\bot^\top \mid z_\bot^\top \in \mathrm{d}(\mathscr{F}) : z \in E\}$*. Then,* $\mathrm{def}(E) \in \Sigma(\mathrm{d}(\mathscr{F}))$.

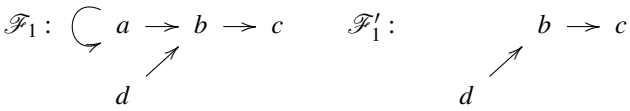
Example 7. *Consider $\mathscr{F}_8$ below. We have:* $\mathrm{co}(\mathscr{F}_8) = \{E_1, E_2\}$, *where* $E_1 = \{\}$, $E_2 = \{b\}$; $\mathrm{def}(E_1) = \{\}$, $\mathrm{def}(E_2) = \{b^b_a\}$; $CO(\mathrm{d}(\mathscr{F}_8)) = \{D_1, D_2\}$, *where* $D_1 = \{\}$, $D_2 = \{b^b_a\}$. *So, it holds that* $\mathrm{def}(E_1) \in CO(\mathrm{d}(\mathscr{F}_8)), \mathrm{def}(E_2) \in CO(\mathrm{d}(\mathscr{F}_8))$.



5.4. Reduct of the attack-defense framework of an AF

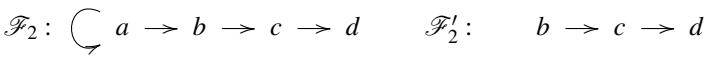
Given the attack-defense framework of an AF, some of the unsatisfiable attack-defenses can be removed, resulting a reduct of the attack-defense framework.

Example 8. *Consider $\mathscr{F}_1$ again. In $\mathscr{F}_1$, for argument c, there are two defenses c^d_b and c^a_b. Intuitively, the defense c^a_b does not contribute to the acceptance of c, because the self-attacking argument a cannot be accepted in any situation and therefore cannot provide support to the acceptance of some other arguments. In other words, when defenses are used to evaluate the status of arguments, the status of c can be determined only according to defense c^d_b, without considering unstatisfiable defenses related to the self-attacking arguments. According to Theorem 7,* $\mathrm{d}(\mathscr{F}_1) = \{a^a_a, b^a_a, c^a_b, c^d_b, d^\top_\perp\}$. *The reduct of* $\mathrm{d}(\mathscr{F}_1)$ *is* $\mathrm{d}(\mathscr{F}_1)^- = \{c^d_b, d^\top_\perp\}$. *Under any semantics* $\Sigma \in \{CO, PR, GR\}$, $\Sigma(\mathrm{d}(\mathscr{F}_1)) = \Sigma(\mathrm{d}(\mathscr{F}_1)^-) = \{D\}$, *where* $D = \{c^d_b, d^\top_\perp\}$.

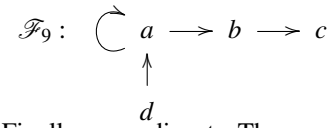


It is interesting to note that while it is natural to remove unsatisfiable attack-defenses without affecting the evaluation of some other attack-defenses, odd cycles cannot be removed from an AF without affecting the evaluation of some other arguments, except same special cases. For instance, in $\mathscr{F}_1$, the self-attacking argument a can be removed without affecting the evaluation of other arguments in $\mathscr{F}_1$. In other words, under Dung's semantics $\sigma \in \{co, pr, gr\}$, $\sigma(\mathscr{F}_1) = \sigma(\mathscr{F}'_1)$. However, such removal of self-attacking arguments cannot be applied to all situations in general.

Example 9. *Consider $\mathscr{F}_2$ again. Under Dung's admissibility semantics, no argument is accepted. If we remove the self-attacking argument a, resutling $\mathscr{F}'_2$, then arguments b and d will be accepted. This means that the removal of a self-attacking argument affects the status of other arguments. However, according to Theorem 7, in the set of attack-defenses* $\mathrm{d}(\mathscr{F}_2) = \{a^a_a, b^a_a, c^a_b, d^b_c\}$, a^a_a, b^a_a *and* c^a_b *are unsatisfiable under any semantics. After removing all these unsatisfiable defense, we obtain a reduct of* $\mathrm{d}(\mathscr{F}_2)$, *i.e.,* $\mathrm{d}(\mathscr{F}_2)^- = \{d^b_c\}$. *Under all semantics, both* $\mathrm{d}(\mathscr{F}_2)$ *and* $\mathrm{d}(\mathscr{F}_2)^-$ *have one unique extension, which is the empty set.*

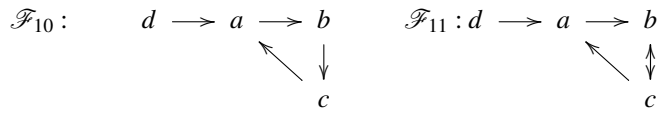


Another point worth to be noted is that the unsatisfiability of an attack-defense does not mean that its defendee is not acceptable. See the following example. In this case, b^a_a is unacceptable, but the argument b is acceptable.

$\mathscr{F}_9$: 

Finally, according to Theorems 6, 7 and 8, the unsatisfiability of attack-defenses of an AF is not relevant to (and therefore not affacted by) the addition of some other arguments or attacks to the AF. The following example further illustrates this property.

Example 10. $\mathrm{d}(\mathscr{F}_{10}) = \{d_\perp^\top, a_d^\perp, a_c^b, b_a^d, b_a^c, c_b^a\}$, *in which* $a_d^\perp$, a_c^b, b_a^c *and* c_b^a *are unacceptable. So,* $\mathrm{d}(\mathscr{F}_{10})^- = \{d_\perp^\top, b_a^d\}$. *After adding an attack from c to b, the attack-defense framework of the resulting AF is* $\mathrm{d}(\mathscr{F}_{11}) = \mathrm{d}(\mathscr{F}_{10}) \cup \{b_c^b, c_b^c\}$, *and* $\mathrm{d}(\mathscr{F}_{11})^- = \mathrm{d}(\mathscr{F}_{10})^- \cup \{b_c^b, c_b^c\}$.



5.5. Equivalence of AFs under attack-defense semantics

The standard equivalence and strong equivalence of AFs cannot represent the equivalence of some AFs in terms of some interesting interpretations as illustrated in Section 1. The attack-defense equivalence provides more information. In this section, we introduce some relations between attack-defense equivalence and the existing two types of equivalence of AFs.

Theorem 14. *Let* $\mathscr{F}$ *and* $\mathscr{G}$ *be two AFs. If* $\mathrm{d}(\mathscr{F}) \equiv_d^\Sigma \mathrm{d}(\mathscr{G})$, *then* $\mathscr{F} \equiv^\sigma \mathscr{G}$, *where* $\Sigma \in \{CO, PR, GR, ST\}$, $\sigma \in \{co, pr, gr, st\}$.

Note that in many cases $\mathscr{F} \equiv^\sigma \mathscr{G}$, but $\mathrm{d}(\mathscr{F}) \not\equiv_d^\Sigma \mathrm{d}(\mathscr{G})$. Consider the following example.

Example 11. *Since* $\mathrm{co}(\mathscr{F}_5) = \mathrm{co}(\mathscr{F}_6) = \{\{a, c\}\}$, *it holds that* $\mathscr{F}_5 \equiv^{\mathrm{co}} \mathscr{F}_6$. *Since* $CO(\mathrm{d}(\mathscr{F}_5)) = \{a_\perp^\top, c_b^a\}$ *and* $CO(\mathrm{d}(\mathscr{F}_6)) = \{a_\perp^\top, c_\perp^\top\}$, $CO(\mathrm{d}(\mathscr{F}_5)) \neq CO(\mathrm{d}(\mathscr{F}_6))$. *So, it is not the case that* $\mathrm{d}(\mathscr{F}_5) \equiv_d^{CO} \mathrm{d}(\mathscr{F}_6)$.

About the relation between attack-defense equivalence and strong equivalence of AFs, due to space limit, we only consider Dung's complete semantics. For further information about strong equivalence and kernels under Dung's semantics, the reader is referred to [11]. Formally, we have the following lemma and theorem.

Lemma 1. *It holds that* $CO(\mathrm{d}(\mathscr{F})) = CO(\mathrm{d}(\mathscr{F}^{\mathrm{ck}}))$.

Theorem 15. *Let* $\mathscr{F}$ *and* $\mathscr{G}$ *be two AFs. If* $\mathscr{F} \equiv_s^{\mathrm{co}} \mathscr{G}$, *then* $\mathrm{d}(\mathscr{F}) \equiv_d^{CO} \mathrm{d}(\mathscr{G})$.

Note that in many cases $\mathrm{d}(\mathscr{F}) \equiv_d^{CO} \mathrm{d}(\mathscr{G})$, but $\mathscr{F} \not\equiv_s^{\mathrm{co}} \mathscr{G}$. Consider the following example.

Example 12. *Since* $CO(\mathrm{d}(\mathscr{F}_{12})) = CO(\mathrm{d}(\mathscr{F}_{13})) = \{\{a_\perp^\top, c_b^a\}\}$, $\mathrm{d}(\mathscr{F}_{12}) \equiv_d^{CO} \mathrm{d}(\mathscr{F}_{13})$. *However, since* $\mathscr{F}_{12}^{\mathrm{ck}} \neq \mathscr{F}_{13}^{\mathrm{ck}}$, $\mathscr{F}_{12} \not\equiv_s^{\mathrm{co}} \mathscr{F}_{13}$.

The relation between root equivalence and Dung's standard equivalence is formulated by the following property.

Theorem 16. *Let* $\mathscr{F} = (\mathscr{A}_1, \rightarrow_1)$ *and* $\mathscr{H} = (\mathscr{A}_2, \rightarrow_2)$ *be two AFs. If* $\mathrm{d}(\mathscr{F}) \equiv_r^{\Sigma} \mathrm{d}(\mathscr{H})$*, then* $\mathscr{F} \equiv^{\sigma} \mathscr{H}$*, where* $\Sigma \in \{\mathrm{CO}, \mathrm{PR}, \mathrm{GR}, \mathrm{ST}\}$ *and* $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}\}$.

Note that in many cases $\mathscr{F} \equiv^{\sigma} \mathscr{H}$, but $\mathrm{d}(\mathscr{F}) \not\equiv_r^{\Sigma} \mathrm{d}(\mathscr{H})$. This can be easily verified by considering $\mathscr{F}_5$ and $\mathscr{F}_6$ in Example 11.

The notion of root equivalence of AFs can be used to capture a kind of summarization in the graphs. Consider the following example borrowed from [3].

Example 13. *Let* $\mathscr{F}_{14} = (\mathscr{A}, \rightarrow)$ *and* $\mathscr{F}_{15} = (\mathscr{A}', \rightarrow')$*, illustrated below. Under complete semantics,* $\mathscr{F}_{15}$ *is a summarization of* $\mathscr{F}_{14}$ *in the sense that* $\mathscr{A}' \subseteq \mathscr{A}$*, and the root reason of each argument in* $\mathscr{F}_{15}$ *is the same as that of each corresponding argument in* $\mathscr{F}_{14}$*. More specifically, it holds that* $root_{CO}(e_3, \mathrm{d}(\mathscr{F}_{14})) = root_{CO}(e_3, \mathrm{d}(\mathscr{F}_{15})) = \{\{\top\}\}$*,* $root_{CO}(e_2, \mathrm{d}(\mathscr{F}_{14})) = root_{CO}(e_2, \mathrm{d}(\mathscr{F}_{15})) = \{\{\top\}\}$*, and* $root_{CO}(e_1, \mathrm{d}(\mathscr{F}_{14})) = root_{CO}(e_1, \mathrm{d}(\mathscr{F}_{15})) = \{\{\top\}\}$.

$\mathscr{F}_{14}$: $e_1 \succ a_1 \succ a_2 \succ o \succ e_3$; $e_2 \succ b_1 \succ b_2$ (with $b_2 \nearrow o$)

$\mathscr{F}_{15}$: $e_1 \succ o \succ e_3$; $e_2 \nearrow o$

Formally, we have the following definition.

Definition 19 (Summarization of AFs)**.** *Let* $\mathscr{F} = (\mathscr{A}_1, \rightarrow_1)$ *and* $\mathscr{H} = (\mathscr{A}_2, \rightarrow_2)$ *be two AFs.* $\mathscr{F}$ *is a summarization of* $\mathscr{H}$ *under a semantics* Σ *iff* $\mathscr{A}_1 \subset \mathscr{A}_2$*, and* $\mathscr{F} \equiv_{r,\mathscr{A}_1}^{\Sigma} \mathscr{H}$.

6. Conclusions

The main contributions of this paper are three-fold. First, we have introduced novel notions of *attack-defense* and *successful attack-defenses*, and used them to formulate an attack-defense framework and attack-defense semantics. It turns out that an attack-defense framework is more expressive than a Dung-style argumentation framework, in the sense that Dung's semantics can be represented in attack-defense semantics, but not vice versa. Second, we have studied three types of unsatisfiable attack-defenses. The attack-defenses related to odd cycles can be removed without affecting the evaluation of other attack-defenses, while odd cyles cannot be removed from an AF without affecting the evaluation of some other arguments in general. Third, we have formulated two new kinds of equivalence relation between AFs, i.e., attack-defense equivalence and root equivalence, and shown that attack-defense semantics can be used to capture the equivalence of AFs from the perspective of reasons for accepting arguments. In addition, we have defined a notion of summarization of AFs by exploiting root equivalence.

The idea of exploiting more usages of attacks has been used in some previous approaches, e.g., [13], [1], and [2]. However, defining a semantics in terms of the notion of attack-defense and studying the properties of this new semantics are novel.

Meanwhile, the notions of attack, defense and acceptability have very closed relations. In this paper, the notion of attack-defense is defined according to Dung's notion of admissibility. In the future work, it is worth to consider other notions of admissibility, e.g., strong admissibilty [6], and weak admissibilty [4].

Last but not least, attack-defense semantics and root equivalence can be applied to dialogues and explainable AI. It would be interesting to investigate new explanation methods by combining attack-defense semantics and some existing approaches, e.g., [8], [5], and [7].

References

[1] Ryuta Arisaka, Jeremie Dauphin, Ken Satoh, and Leendert van der Torre. Multi-agent argumentation and dialogue. FLAP, 9(4):921–954, 2022.

[2] Pietro Baroni, Federico Cerutti, Massimiliano Giacomin, and Giovanni Guida. AFRA: argumentation framework with recursive attacks. Int. J. Approx. Reason., 52(1):19–37, 2011.

[3] Pietro Baroni, Guido Boella, Federico Cerutti, Massimiliano Giacomin, Leendert van der Torre, and Serena Villata. On the input/output behavior of argumentation frameworks. Artificial Intelligence, 217:144–197, 2014.

[4] Ringo Baumann, Gerhard Brewka, and Markus Ulbricht. Shedding new light on the foundations of abstract argumentation: Modularization and weak admissibility. Artif. Intell., 310:103742, 2022.

[5] AnneMarie Borg and Floris Bex. Contrastive explanations for argumentation-based conclusions. In Piotr Faliszewski, Viviana Mascardi, Catherine Pelachaud, and Matthew E. Taylor, editors, 21st International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2022, Auckland, New Zealand, May 9-13, 2022, pages 1551–1553. International Foundation for Autonomous Agents and Multiagent Systems (IFAAMAS), 2022.

[6] Martin Caminada and Sri Harikrishnan. Strong admissibility, a tractable algorithmic approach. In Sarah Alice Gaggl, Jean-Guy Mailly, Matthias Thimm, and Johannes Peter Wallner, editors, Proceedings of the Fourth International Workshop on Systems and Algorithms for Formal Argumentation co-located with the 9th International Conference on Computational Models of Argument (COMMA 2022), Cardiff, Wales, United Kingdom, September 13, 2022, volume 3236 of CEUR Workshop Proceedings, pages 33–44. CEUR-WS.org, 2022.

[7] Kristijonas Cyras, Antonio Rago, Emanuele Albini, Pietro Baroni, and Francesca Toni. Argumentative XAI: A survey. In Zhi-Hua Zhou, editor, Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021, Virtual Event / Montreal, Canada, 19-27 August 2021, pages 4392–4399. ijcai.org, 2021.

[8] Sylvie Doutre, The o Duchatelle, and Marie-Christine Lagasquie-Schiex. Visual explanations for defence in abstract argumentation. In Noa Agmon, Bo An, Alessandro Ricci, and William Yeoh, editors, Proceedings of the 2023 International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2023, London, United Kingdom, 29 May 2023 - 2 June 2023, pages 2346–2348. ACM, 2023.

[9] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence, 77(2):321–358, 1995.

[10] Beishui Liao and Leendert van der Torre. Explanation semantics for abstract argumentation. In Henry Prakken, Stefano Bistarelli, Francesco Santini, and Carlo Taticchi, editors, Computational Models of Argument - Proceedings of COMMA 2020, Perugia, Italy, September 4-11, 2020, volume 326 of Frontiers in Artificial Intelligence and Applications, pages 271–282. IOS Press, 2020.

[11] Emilia Oikarinen and Stefan Woltran. Characterizing strong equivalence for argumentation frameworks. In Principles of Knowledge Representation and Reasoning: Proceedings of the Twelfth International Conference, KR 2010, Toronto, Ontario, Canada, May 9-13, 2010, 2010.

[12] Pietro Baroni, Dov Gabbay, Massimiliano Giacomin, and Leendert van der Torre, editors. Handbook of Formal Argumentation. College Publications, 2018.

[13] Serena Villata, Guido Boella, and Leendert W. N. van der Torre. Attack semantics for abstract argumentation. In Toby Walsh, editor, IJCAI 2011, Proceedings of the 22nd International Joint Conference on Artificial Intelligence, Barcelona, Catalonia, Spain, July 16-22, 2011, pages 406–413. IJCAI/AAAI, 2011.