

Diagnosis of Coupled Quantised Systems Based on Automata Networks

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1 Introduction

Quantised systems are continuous-variable systems, which signals can only be measured discretely through quantisers. The input-output-behaviour of such systems is characterised by sequences of symbols or discrete values.

In previous works it has been shown that faults in quantised systems can be diagnosed using a purely discrete model in form of a stochastic automaton [2]. The behavioural relation of the automaton can be determined automatically from a given state space model of the continuous system. The stochastic automaton has proven to be a type of model both powerful and simple to use. However, the drawback of this type of model is that while being simple it can become too large to handle, especially for coupled and complex systems. This is known in discrete-event system theory as *state space explosion*.

The project aims at reducing the complexity of the model of the coupled quantised system by component-oriented modelling, where each subsystem is represented by a stochastic automaton. The overall model is, hence, an automata network, which will then be used for fault diagnosis [1].

2 The automata network

An automata network consists of several stochastic automata which are interconnected. Fig. 1 shows the network is not limited to parallel systems, but can include serial and feedback connections. The example illustrates that the coupling signals need not be measurable and that each automaton can have multiple inputs and outputs. This universality is necessary to model existing industrial applications.

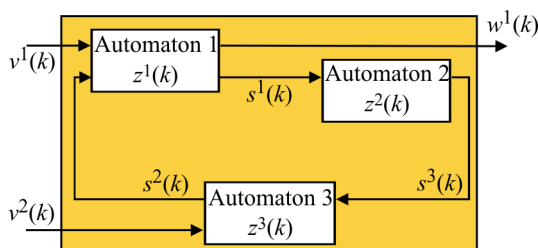


Figure 1: Automata network

A physical system usually consists of many components which interact with each other. While the existing modelling approach

did not take this structure into consideration, in this project every component will be modelled separately resulting in multiple stochastic automata which will be interconnected according to the physical system. It is clear that this approach demands fewer resources, since independencies between the components are regarded automatically.

3 Modelling the network

The i th stochastic automaton of the network is described by the tuple $A_i(\mathcal{N}_{z_i}, \mathcal{N}_{v_i}, \mathcal{N}_{w_i}, L_i, z_{i0})$, where $\mathcal{N}_{z_i} = \{1, 2, \dots, N_i\}$ is the set of states z_i , $\mathcal{N}_{v_i} = \{1, 2, \dots, M_i\}$ the set of inputs v_i , $\mathcal{N}_{w_i} = \{1, 2, \dots, R_i\}$ the set of outputs w_i and z_{i0} is the initial condition of the automaton A_i . The behavioural relation

$$L_i = P(z_{ip}(k+1) = z_i(k+1), w_{ip}(k) = w_i(k) | z_{ip}(k) = z_i(k), v_{ip}(k) = v_i(k)) \quad (1)$$

describes the dynamics of the automaton. Items with the index p are stochastic variables.

Automata may be connected in three different ways, namely parallel, serial, and as a feedback connection (Fig. 2). For the supervision of the overall process (marked by the grey boxes in Fig. 2) it is necessary to analyse how the different automata influence each other or in other words how the stochastic dependencies of the partial models influences the overall behaviour. The combination of partial models to get the overall model is called *composition*. However, the aim of this project is not to calculate the overall model, since this would destroy all the benefits of the network representation, but to use composition rules to calculate the overall behaviour directly.

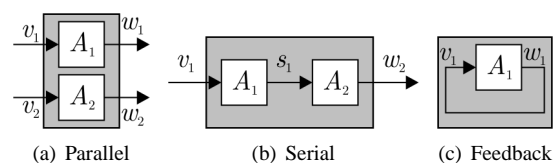


Figure 2: Connection types

The following composition rules show the complexity reduction of the system representation:

Parallel composition: Two parallel automata can be composed as follows:

$$\begin{aligned} \tilde{L}\left(\left(\begin{matrix} z'_1 \\ z'_2 \end{matrix}\right), \left(\begin{matrix} w_1 \\ w_2 \end{matrix}\right) \mid \left(\begin{matrix} z_1 \\ z_2 \end{matrix}\right), \left(\begin{matrix} v_1 \\ v_2 \end{matrix}\right)\right) &= \tilde{L}(z', \tilde{w} \mid \tilde{z}, \tilde{v}) \\ &= L_1(z'_1, w_1 \mid z_1, v_1) \cdot L_2(z'_2, w_2 \mid z_2, v_2). \end{aligned} \quad (2)$$

Serial composition: Two automata in a series connection can be composed as follows:

$$\tilde{L}(z', w_2 \mid \tilde{z}, v_1) = \sum_{s_1} L_1(z'_1, s_1 \mid z_1, v_1) \cdot L_2(z'_2, w_2 \mid z_2, s_1). \quad (3)$$

Feedback composition: An automaton with a feedback connection can be transformed to an autonomous automaton as follows:

$$\tilde{L}(z' \mid \tilde{z}) = \sum_{w_1} L_1(z'_1, w_1 \mid z_1, w_1). \quad (4)$$

4 State Observation

The aim of state observation is to reconstruct the current state of the system of interest from input/output measurement sequences. A model of the system in form of an automata network is created and the state of the model is observed. Since the model is guaranteed to be complete (model behaviour includes the system behaviour) the observation result includes the current state of the system. Because the model is nondeterministic usually the result is not unambiguous, but instead a probability distribution over the state set is obtained [2].

As opposed to the solved observation problem of a single stochastic automaton the observation of the automata network has shown to be far more difficult. In case of parallel automata (see Fig. 2) it has been proven that each automaton can be observed independently. The overall probability distribution p of the system can be calculated from the distributions p_i of the single automata using tensor algebra:

$$\begin{aligned} p(\tilde{z}(k) \mid \tilde{V}(0 \dots k-1), \tilde{W}(0 \dots k-1)) &= \\ \frac{L_1(k-1)p_1(k-2) \otimes L_2(k-1)p_2(k-2)}{\sum_{z_1(k)} L_1(k-1)p_1(k-2) \otimes \sum_{z_2(k)} L_2(k-1)p_2(k-2)}. \end{aligned} \quad (5)$$

This calculation uses far less resources than the observation with a single large model. However, in case of the serial and feedback connection the overall probability distribution cannot be calculated from the single distributions p_i since usually the automata are not stochastically independent. Here the overall distribution has to be calculated directly using the composition rules.

5 Diagnosis

The aim of diagnosis is to find the fault f which causes the plant to work outside the nominal operating point. Since this makes it necessary to find the current operating point, the diagnosis task always includes an observation task.

To solve the observation task the automaton state is then extended to include the fault symbols:

$$\hat{z}_i := \begin{pmatrix} z_i \\ f_i \end{pmatrix}. \quad (6)$$

Then the extended state \hat{z} is observed using the observation algorithm. The diagnosis result is derived by projecting the calculated probability distribution onto the fault space.

6 Experimental example

The experimental plant that will be used throughout the project is the small-scale model of a power plant (Fig. 3). It consists of two identical generators each driven by a turbine propelled by air pressure. Each generator can produce power up to 300 W. The generators can be operated independently or, after synchronisation, as one unit. Strictly speaking, since both turbines are connected to the same air-supply they are not completely independent.

This plant is suitable for the project for several reasons. First this plant is already complex enough to result in an unstructured automaton model which is too large to handle with today's computers since the behavioural relation contains approximately 10^{11} transitions. Second fault scenarios can easily be experimented. Additionally, since the plant consists of several parts, which can be modelled separately, it is a perfect testbed for the modelling, observation, and diagnosis of automata networks.

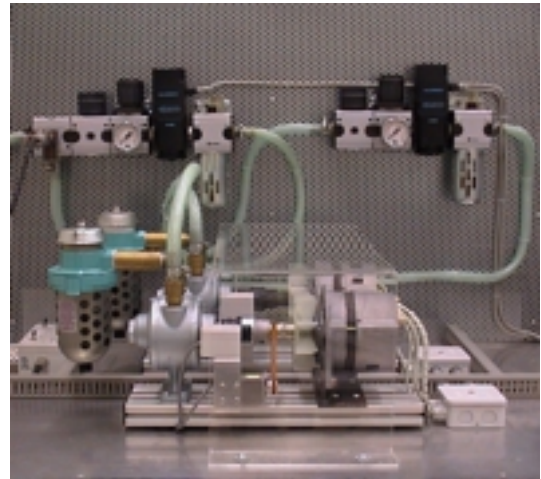


Figure 3: Physical model of a power plant

References

- [1] J. Lunze and J. Schröder. Fault diagnosis of stochastic automata networks. In *Safeprocess*, pages 825–830, Washington, 2003.
- [2] J. Schröder. *Modelling, State Observation and Diagnosis of Quantised Systems*. Springer-Verlag, Berlin, 2003.